New GFT Models in 3 and 4 Dimensions

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- Work done in collaboration with Daniele Oriti (ITF, University of Utrecht).

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- ‘A New Class of Group Field Theories for Discrete 1st Order Quantum Gravity’, [arXiv: [gr-qc] 0710.2679]

Plan of the talk

- Motivations for the new models
- Construction of the new models
- Properties of the new models
- Summary

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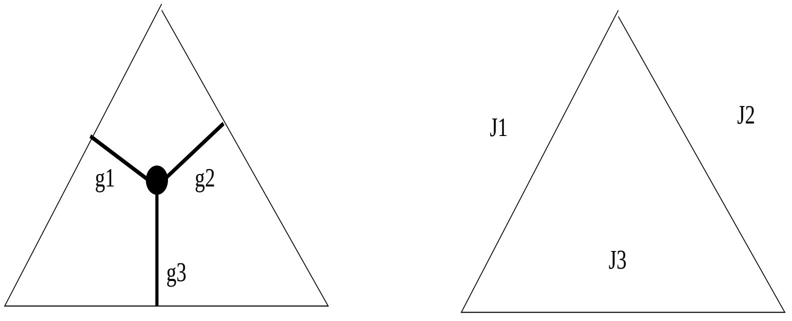

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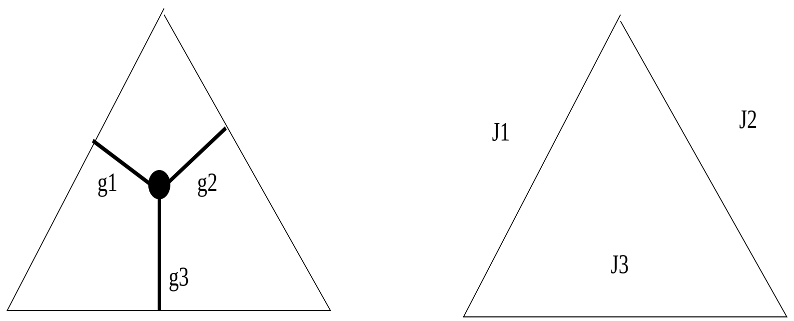

- Why bother with two fields?

Motivations contd.

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- Existing GFTs give amplitudes where one of the variables has already been implicitly integrated over. Thus the nice exponent of the action is integrated over and no longer appears explicitly in the amplitudes.
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- Existing GFTs are a-causal in the sense that they provide a covariant path integral realization of the Hamiltonian constraint.

Motivations concluded

The usual models give

$$\begin{aligned}\delta(\mathcal{H}) &= \int_{-\infty}^{\infty} (\mathcal{D}N \ldots)\, e^{i(N\mathcal{H}+\ldots)} \\ &= \int_{det(B)>0} \mathcal{D}A\mathcal{D}B\, e^{iS[A,B]} + \int_{det(B)<0} \mathcal{D}A\mathcal{D}B\, e^{iS[A,B]}\end{aligned}$$

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- Other reasons (more direct implementation of simplicity constraints, analysis of the translational symmetry at the GFT level, etc . . .)

The new GFT

- The field is a function of 3 SU(2) group elements and three $\mathfrak{su}(2)$ Lie algebra elements, $\phi(g_1, g_2, g_3; B_1, B_2, B_3)$. The g's are interpreted as usual as being the parallel transports connecting the center of the triangle with the edges, while the B's are interpreted as being the B field discretized on the edges.

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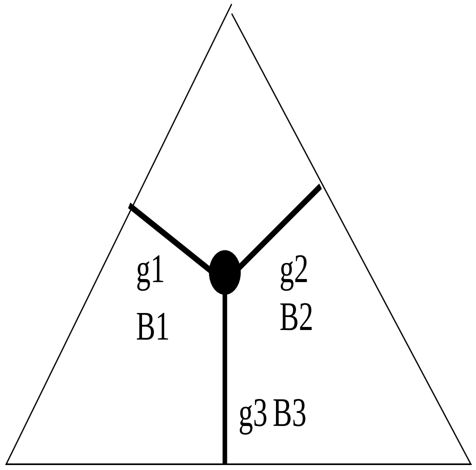

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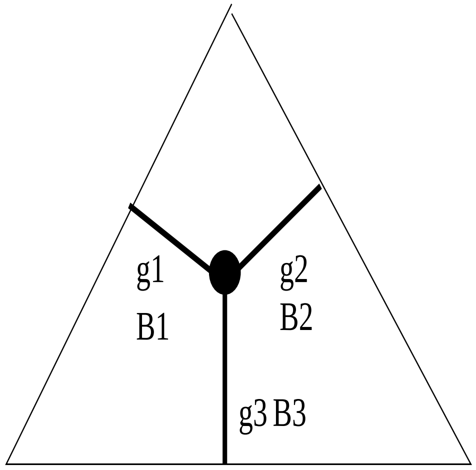

- The rotational gauge-invariance is implemented via $\phi(g_1 h, g_2 h, g_3 h; hB_1 h^{-1}, hB_2 h^{-1}, hB_3 h^{-1}) = \phi(g_1, g_2, g_3; B_1, B_2, B_3)$

The new GFT construction contd.

- The GFT action is given by

$$S = \int \phi \left(\prod_{i=1}^{3} (\Box_i - B_i^2) \right) \phi + \lambda \int \phi\, \phi\, \phi\, \phi$$

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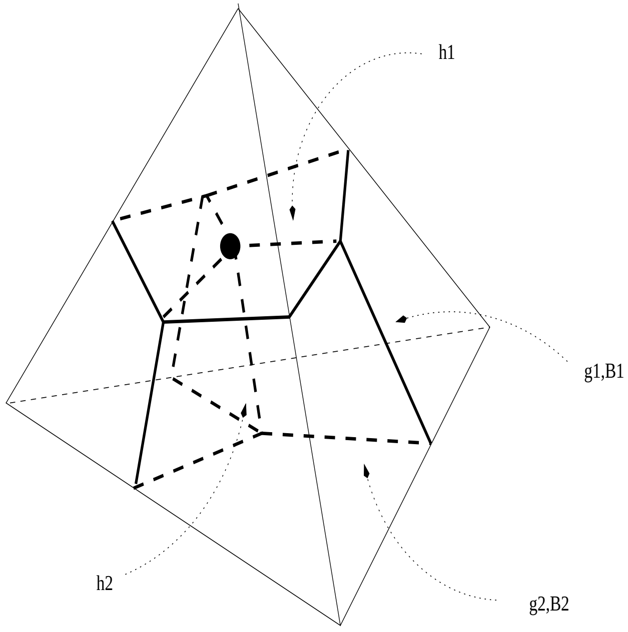
- The B's are squared using the Killing form on $\mathfrak{su}(2)$.
- The pattern of contraction of the variables in the vertex is exactly the same as in the usual models (the B's contract in the same way as the g's).
- The kinetic term is arrived at by noting that from the canonical analysis $B \sim \frac{d}{dg} \sim J$.

The new GFT construction concluded



$$\delta(g_1 h_1 h_2^{-1} g_2^{-1})$$

$$\delta(B_2 - (h_2 h_1^{-1}) B_1 (h_2 h_1^{-1})^{-1})$$

The new GFT properties

- We quantize this model now using path integrals and expand in Feynman graphs

$$Z = \int \mathcal{D}\phi e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{|\Gamma|}}{\text{sym}(\Gamma)} Z_{\Gamma}$$

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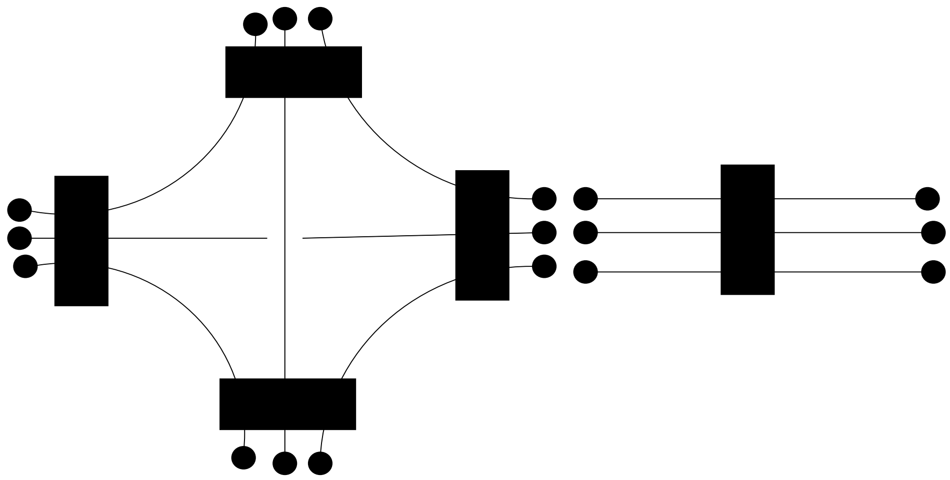
- We extract from the action on the previous slide the propagator and the vertex amplitude (the two ingredients needed to construct the Feynman graphs).



The new GFT *properties contd.*

- The Feynman amplitude for Z_Γ factorizes per edge/dual face of the triangulation/dual 2-complex. In other words

$$Z_\Gamma = \int \left(\prod_e dB_e \prod_{e*} dh_{e^*} \right) \prod_e A_e[B_e, H_e, N_e]$$

where B_e is the B-field associated to the edge e, H_e is the holonomy around the dual face and N_e is the number of dual vertices in the dual face.

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$$A[B, H, N] = \delta(B - HBH^{-1})\, \mu(H, B, N)\, e^{iS_c[B,H,N]}\, e^{iS_R[B,H]}$$

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- The amplitude is made out 4 pieces : the gauge-fixing term, the measure, the quantum corrections, the Regge/BF action.

The new GFT *properties contd.*

- The Regge term is $e^{iS_R[B,H]}$, where

$$S_R[B,H] = |B||\theta(H)|$$

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- The interpretation of the B variables coming from the amplitudes matches the one we started with.
- Taking into account the gauge-fixing term $\delta(B - HBH^{-1})$, one sees that the B field is aligned with the F = ln(H), and one can rewrite $S_R[B,H]$ as

$$S_R[B,H] = \mathrm{Tr}(BF)$$

with a causality restriction (B and F are aligned not anti-aligned).

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- The model is automatically causal.
- The quantum corrections term depends on B and H=e^F in exactly the same way as the Regge/BF term (i.e. through the product $|B||\theta(H)|$ or equivalently via Tr(BF)).
- Since this is just a discretization of the Einstein-Hilbert action, it is natural to think about it as being a discrete analogue of the Ricci scalar R.

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  - In the small length/large number of tetrahedra (continuum) limit $S \sim \int R + o(R^2)$.
- Finally, the measure piece shows that the configurations with large number of tetrahedra of small size are the dominant ones (and the small size tetrahedra become more dominant the more there are). These configurations are the ones most relevant for the continuum limit.

The new GFT ***properties concluded***

- (Unexpected) All the above results carry effortlessly into the Lorentzian regime, with an unexpected byproduct that the model automatically suppresses the configurations (which are present due to the 1st order nature of the theory) which do not have a classical geometric interpretation.

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- In other words, those configurations in which we have a geometric mismatch between the B field and the holonomy (timelike-boost or spacelike-rotation) are exponentially suppressed.
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  When the configuration is non-geometrical (classically) $e^{iS} \to e^{-S}$.
- All the above results also carry over into the 4d case (Riemannian and Lorentzian) which doesn't present any new difficulties, apart from the fact that the gauge-fixing term has to be made slightly stronger.

Summary

- We have seen how to construct a GFT for 1st order theories.
- The obtained amplitudes have the explicit form of an exponent of the Regge/BF action.
- The amplitudes are automatically causal.
- The amplitudes contain corrections to the Regge/BF action.
- The amplitudes strongly favour the configurations which are most suited to describe the continuum limit.
- The amplitudes correctly incorporate the interplay between the connection and the curvature in a 1st order Lorentzian theory.
- and more.