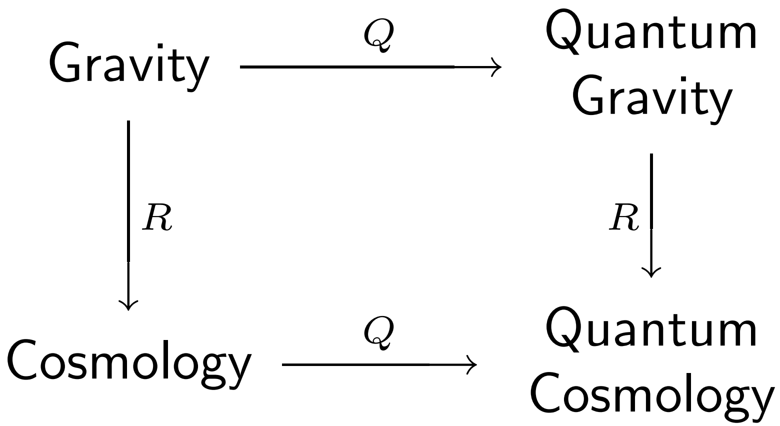
On the Configuration Spaces of Homogeneous Loop Quantum Cosmology and Loop Quantum Gravity

Christian Fleischhack

Department Mathematik
Universität Hamburg

Zakopane, March 2008

1 Motivation



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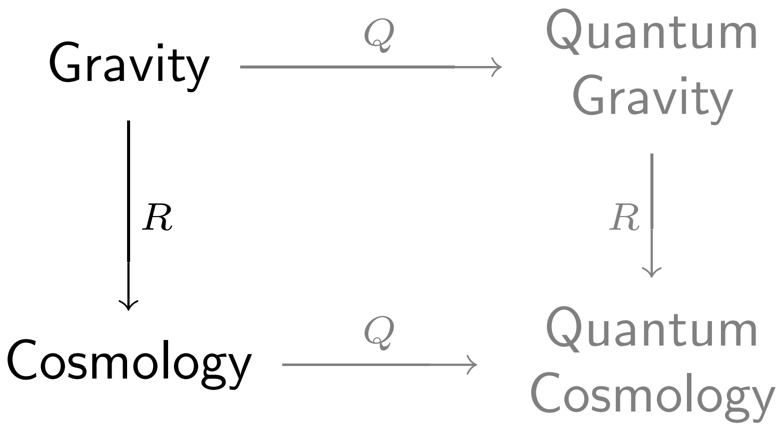

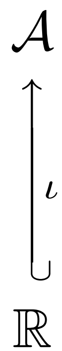

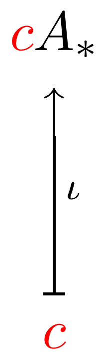

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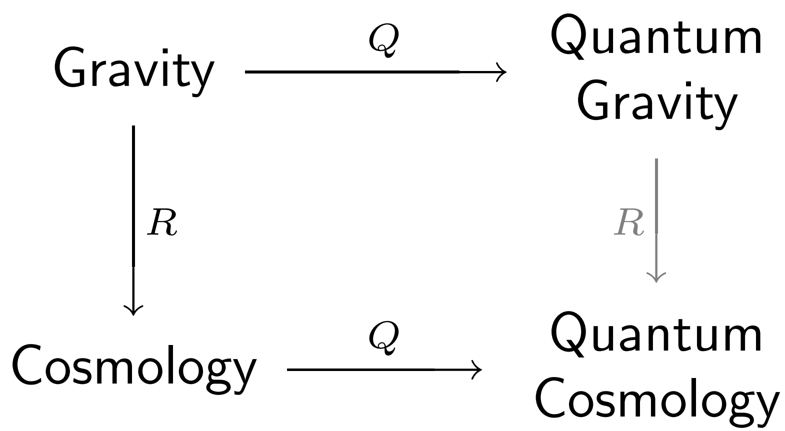

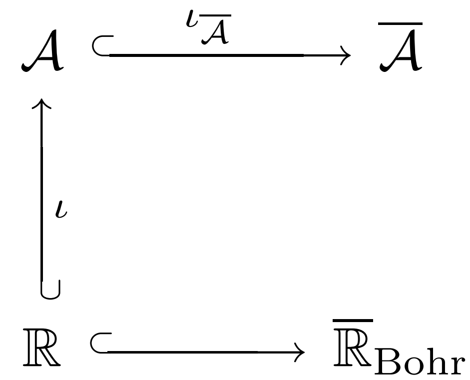

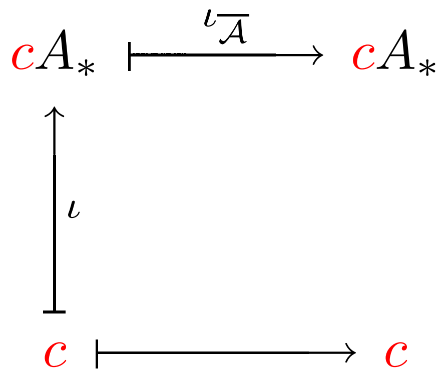

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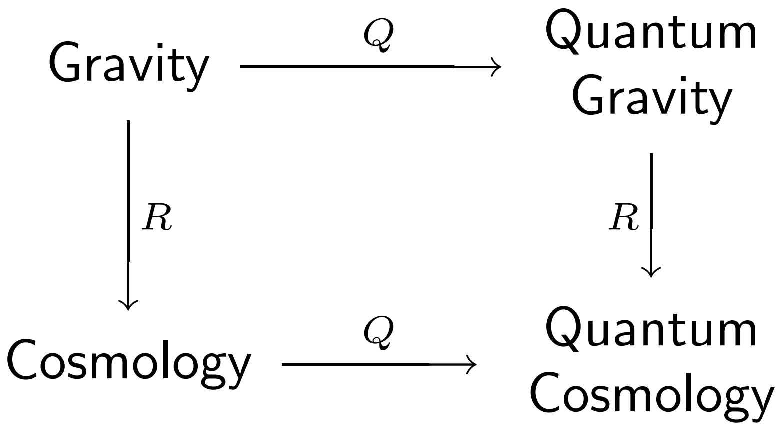

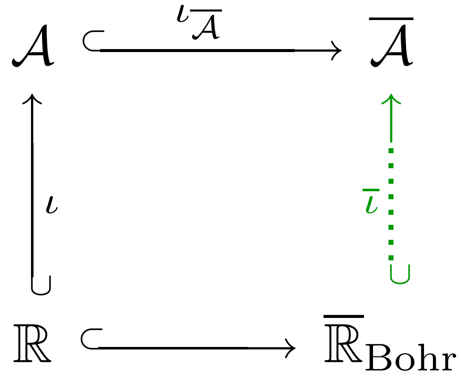

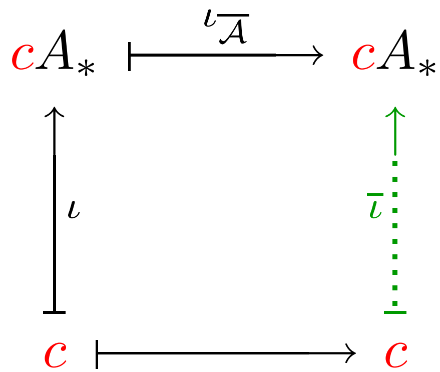

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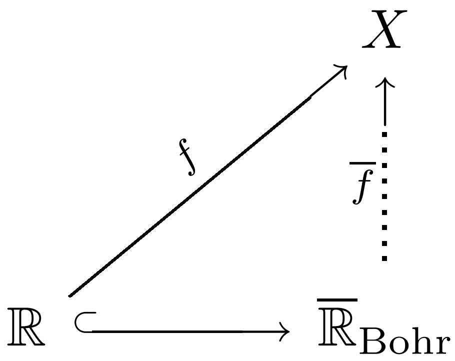

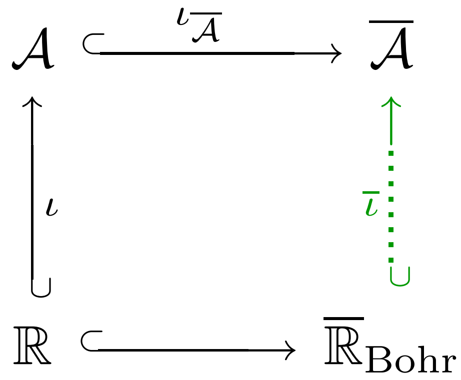

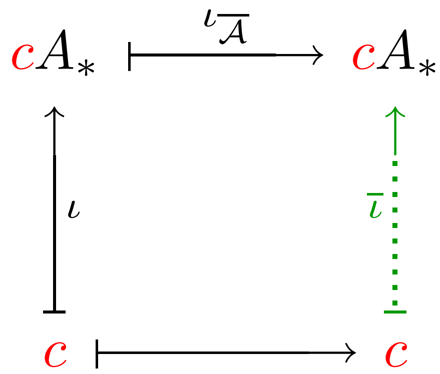

Theorem:

f continuous w.r.t. Bohr topology
$\iff$ f almost periodic

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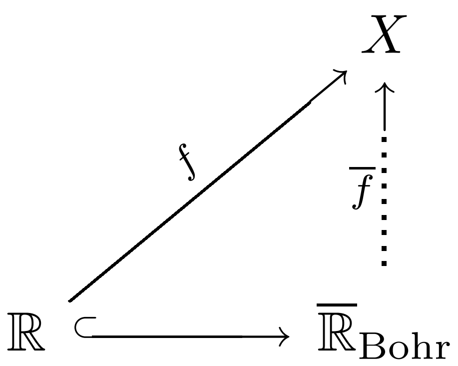

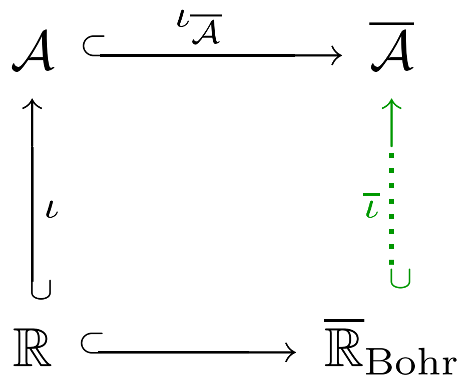

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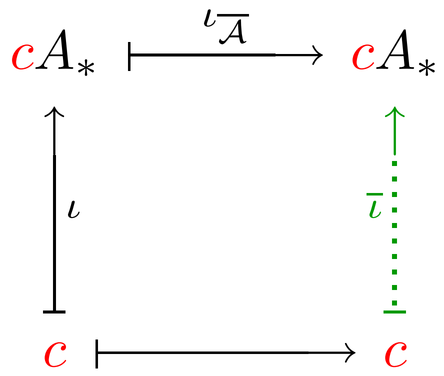

Corollary:

$\bar{\iota}$ exists
$\iff$ $\iota_{\overline{\mathcal{A}}} \circ \iota$ continuous

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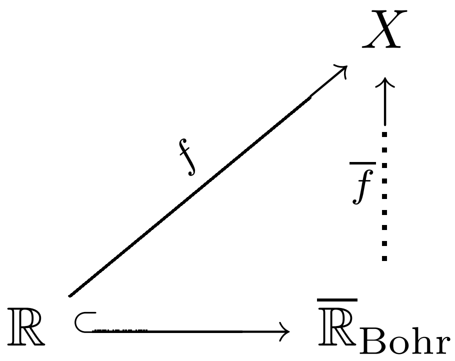

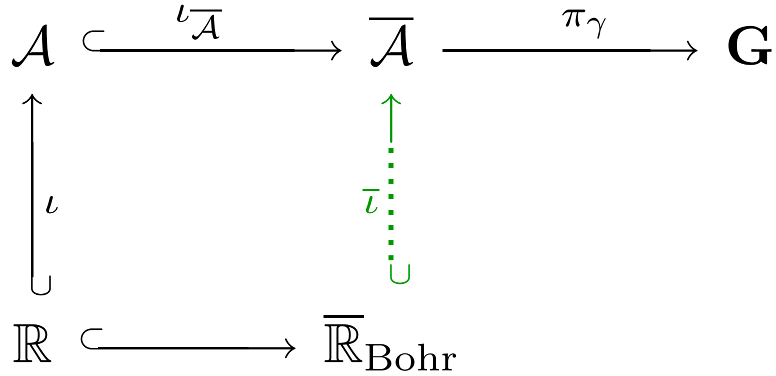

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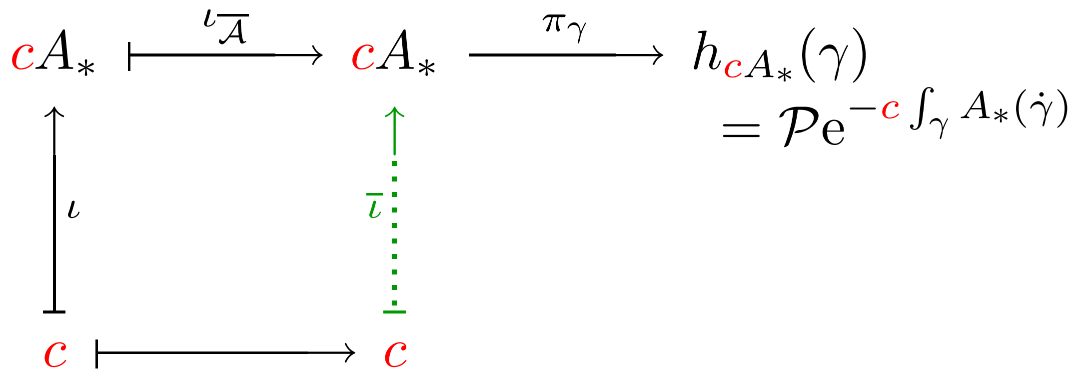

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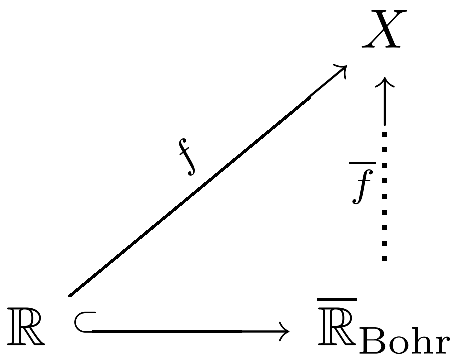

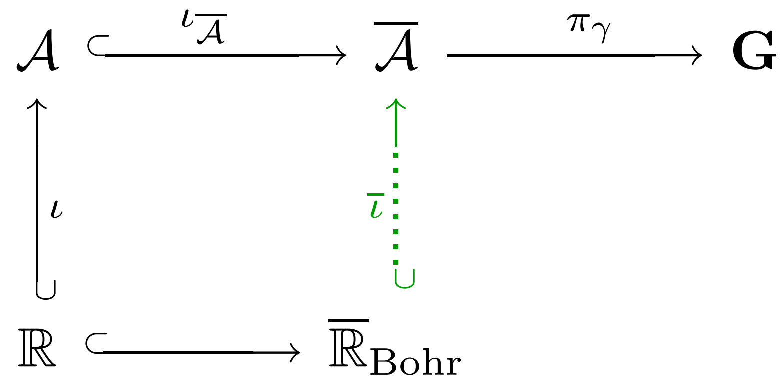

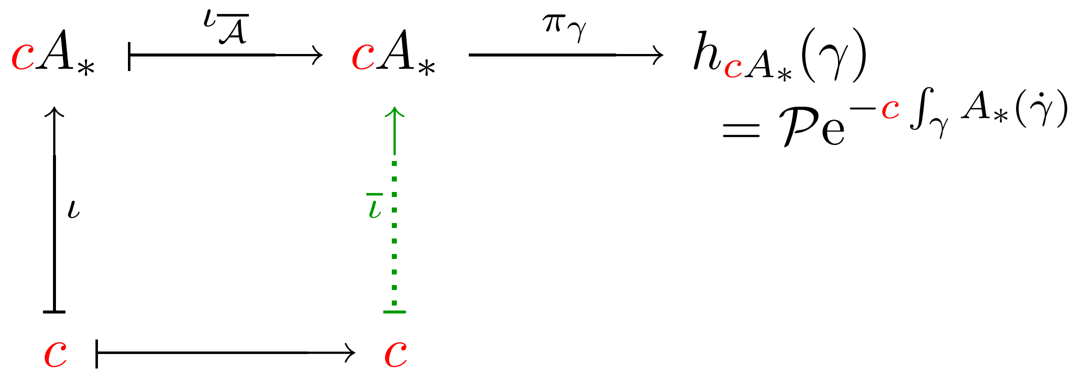

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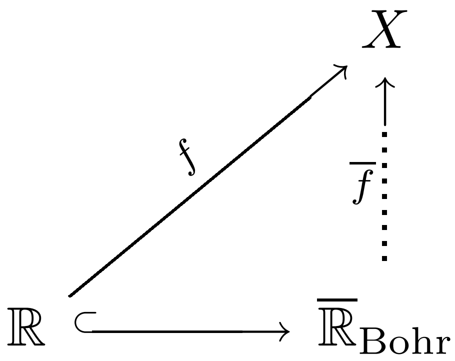

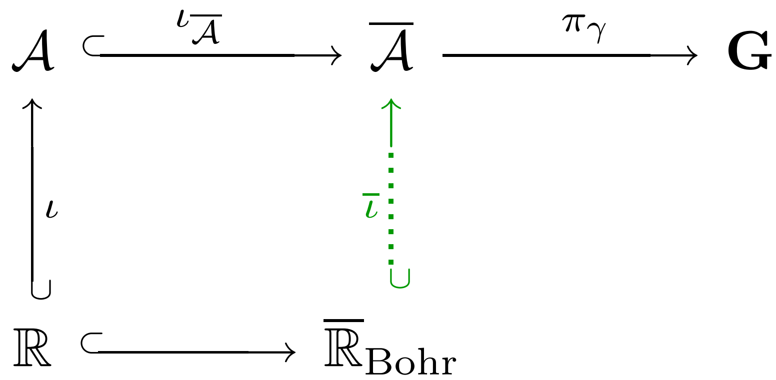

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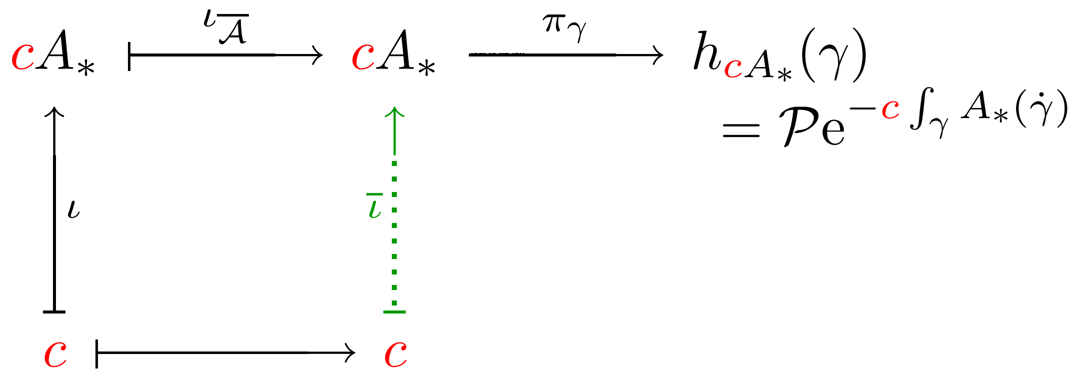

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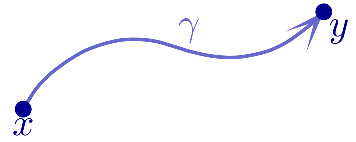
- Parallel Transport



$$g(t) \quad := \quad h_{cA_*}\big(\gamma|_{[0,t]}\big)$$

- Parallel Transport Equation

$$\begin{aligned} \dot{g}(t) \quad &= \quad -c\,A_*\big(\dot{\gamma}(t)\big)\,g(t) \\ g(0) \quad &= \quad \mathbf{1} \end{aligned}$$

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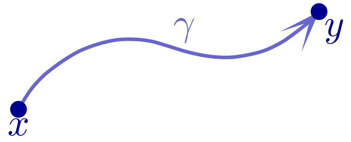

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- Homogeneous Isotropic Connection $\qquad (M = \mathbb{R}^3, \mathbf{G} = SU(2))$

$$A_* \quad = \quad \tau_1 \mathrm{d}x + \tau_2 \mathrm{d}y + \tau_3 \mathrm{d}z$$

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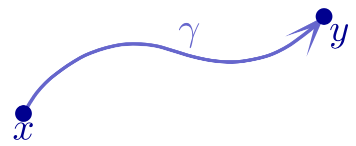

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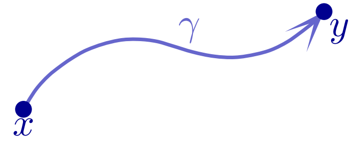

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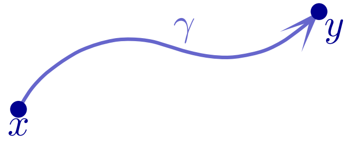

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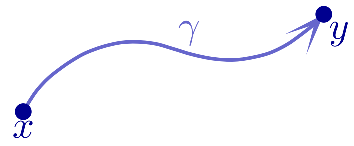

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$$\begin{pmatrix} \dot{a} & \dot{b} \\ -\dot{\overline{b}} & \dot{\overline{a}} \end{pmatrix} = \mathrm{i}c \begin{pmatrix} n & m \\ \overline{m} & -n \end{pmatrix} \begin{pmatrix} a & b \\ -\overline{b} & \overline{a} \end{pmatrix}$$

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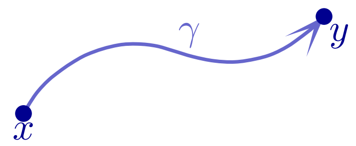

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$$\begin{aligned} \dot{a} &= \mathrm{i}c(na - m\overline{b}) \qquad & a(0) &= 1 \\ \dot{b} &= \mathrm{i}c(nb + m\overline{a}) \qquad & b(0) &= 0 \end{aligned}$$

3 Examples

Question: For which γ is $c \longmapsto h_{cA_*}(\gamma)$ almost periodic?

$$\begin{aligned} \dot{a} &= \mathrm{i}c(na - m\overline{b}) \\ \dot{b} &= \mathrm{i}c(nb + m\overline{a}) \end{aligned}$$

- Second-order Equation

$$\ddot{a} = \mathrm{i}c(\dot{n} - Mn)a - c^2 a + M\dot{a} \qquad \text{with} \quad M := \frac{\dot{m}}{m}$$

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	Straight Line	
$\dot m$ n, $\dot n$ Equations Initial Values $(m(0) = -\mathrm{i})$		
Solution		

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$\dot m$	0	
n, $\dot n$	0	
Equations	$\ddot a + c^2 a = 0$ $\ddot b + c^2 b = 0$	
Initial Values $(m(0) = -\mathrm{i})$	$b(0) = 0$ $\dot b(0) = c$	
Solution		

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	Straight Line	Planar Circle
$\dot{m}$	0	$2\mathrm{i}m$
n, $\dot{n}$	0	0
Equations	$\ddot{a} + c^2 a = 0$ $\ddot{b} + c^2 b = 0$	
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Initial Values $(m(0) = -\mathrm{i})$	$b(0) = 0$ $\dot b(0) = c$	$b(0) = 0$ $\dot b(0) = c$
Solution	$b(t) = \sin ct$ Good!	$b(t) = \frac{\mathrm{e}^{\mathrm{i}t}}{\sqrt{1+\frac{1}{c^2}}} \sin \sqrt{1 + \frac{1}{c^2}}\; ct$

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Equations	$\ddot{a} + c^2 a = 0$ $\ddot{b} + c^2 b = 0$	$\ddot{a} + c^2 a = 2\mathrm{i}m\dot{a}$ $\ddot{b} + c^2 b = 2\mathrm{i}m\dot{b}$
Initial Values $(m(0) = -\mathrm{i})$	$b(0) = 0$ $\dot{b}(0) = c$	$b(0) = 0$ $\dot{b}(0) = c$
Solution	$b(t) = \sin ct$ Good!	$b(t) = \frac{\mathrm{e}^{\mathrm{i}t}}{\sqrt{1+\frac{1}{c^2}}} \sin \sqrt{1 + \frac{1}{c^2}}\; ct$ Well . . .

3 Examples

Question: For which γ is $c \longmapsto h_{cA_*}(\gamma)$ almost periodic?

$$\begin{aligned} \dot a &= \mathrm{i}c(na - m\overline{b}) \\ \dot b &= \mathrm{i}c(nb + m\overline{a}) \end{aligned}$$

- Second-order Equation

$$\ddot a = \mathrm{i}c(\dot n - Mn)a - c^2 a + M\dot a \qquad \text{with} \quad M := \frac{\dot m}{m}$$

- Examples

	Straight Line	Planar Circle
$\dot m$	0	$2\mathrm{i}m$
n, $\dot n$	0	0
Equations	$\ddot a + c^2 a = 0$ $\ddot b + c^2 b = 0$	$\ddot a + c^2 a = 2\mathrm{i}m\dot a$ $\ddot b + c^2 b = 2\mathrm{i}m\dot b$
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Theorem: $\iota : \mathbb{R} \longrightarrow \mathcal{A}$ cannot be continuously extended to $\overline{\iota} : \overline{\mathbb{R}}_{\mathrm{Bohr}} \longrightarrow \overline{\mathcal{A}}$.

4 Non-Almost Periodicity of Parallel Transports

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Definition: t almost periodic $\iff$ $a(t)$ and $b(t)$ almost periodic.

Corollary: t almost periodic $\implies$ $F(t) := \left(\overline{m}a^2 + 2na\overline{b} - m\overline{b}^2\right)\Big|_0^t = 0$

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Proposition: Let there exist an accumulation point of almost periodic t.
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Proof: Assume t almost periodic with $\dot{m}(t) \neq 0$. Then $\dot{m} \neq 0$ on some open $U \ni t$.

$$\text{Assumption} \Longrightarrow F \equiv 0 \Longrightarrow \begin{array}{rcl} \dot{\overline{m}}a^2 + 2\dot{n}a\overline{b} - \dot{m}\overline{b}^2 & = & \dot{F} \equiv 0 \\ \overline{m}a^2 + 2na\overline{b} - m\overline{b}^2 & = & m(0) \end{array}$$

$\Longrightarrow$ a and b independent of c on U; as well as $\dot{a}$ and $\dot{b}$

$$\Longrightarrow na - m\overline{b} = 0 = nb + m\overline{a}$$

$$\Longrightarrow a = (n^2 + \overline{m}m)a = nm\overline{b} + \overline{m}ma = 0 \quad ⚡$$

Thus $\dot{m}(t) = 0$ for all almost periodic t $\longrightarrow$ $\dot{m} \equiv 0$ $\longrightarrow$ $\dot{n} \equiv 0$. **qed**

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Lemma: We have $\lim_{c\to\infty}\bigl(\overline{m}a^2 + 2na\bar b - m\bar b^2\bigr)\big|_0^t = 0.$

Proof: Recall $\ddot a = \mathrm{i}c(\dot n - Mn)a - c^2 a + M\dot a$, and observe, with $\overline{m}m + n^2 = 1$,

$$\overline{m}a^2 + 2na\bar b - m\bar b^2 = \frac{1}{m}\Bigl(-(na - m\bar b)^2 + a^2\Bigr) = \frac{1}{m}\Bigl(\frac{\dot a^2}{c^2} + a^2\Bigr).$$

Use

$$a =: \sqrt{m}\,\alpha \implies \ddot\alpha + c^2\alpha = \bigl(\tfrac14 M^2 - \tfrac12 \dot M + \mathrm{i}c(\dot n - Mn)\bigr)\alpha =: \rho\alpha$$

$$\implies \frac{\mathrm{d}}{\mathrm{d}t}\Bigl(\frac{\dot\alpha^2}{c^2} + \alpha^2\Bigr) = \frac{2}{c^2}(\ddot\alpha + c^2\alpha)\dot\alpha = \frac{\rho}{c^2}\frac{\mathrm{d}}{\mathrm{d}t}\alpha^2.$$

Now,

$$\Bigl(\frac{\dot\alpha^2}{c^2} + \alpha^2\Bigr)\Big|_0^t = -\int_0^t \frac{\dot\rho}{c^2}\,\alpha^2\,\mathrm{d}\tau + \frac{\rho}{c^2}\,\alpha^2\Big|_0^t \xrightarrow{c\to\infty} 0$$

and

$$\frac{1}{m}\Bigl(\frac{\dot a^2}{c^2} + a^2\Bigr)\Big|_0^t = \Bigl(\frac{\dot\alpha^2}{c^2} + \alpha^2\Bigr)\Big|_0^t + \frac{Ma}{mc^2}\Bigl(\dot a - \frac14 Ma\Bigr)\Big|_0^t \xrightarrow{c\to\infty} 0.$$

qed

5 Conclusions

- **Theorem:** Let γ be an analytic curve that is not part of a straight line. Then there is some $T > 0$, such that the parallel transport along $\gamma|_{[0,t]}$ is not almost periodic w.r.t. c for any $0 < t < T$.

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- Open Questions:
  - almost periodicity "by chance"?
  - asymptotic almost periodicity
  - $k = \pm 1$
  - general Lie groups
  - $\ldots$