OBSERVABLES

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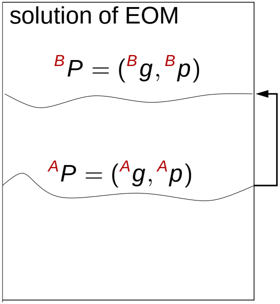
OVERVIEW

1. Dirac observables and how to compute them
2. Causality properties
3. Gauge invariant perturbations around symmetry reduced sectors
4. The problem of clocks (a posteriori motivation and discussion)

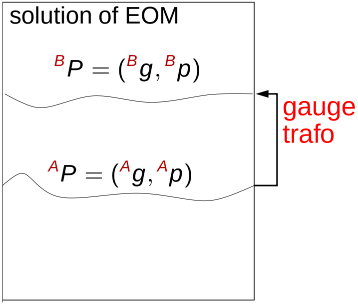
What are Dirac observables?

OBSERVABLES AND DIRAC OBSERVABLES



- observables characterize solutions:
  F: space of solutions $\quad\rightarrow\quad$ **C**
- no explicite parametrization of this space available
- phase space:
  - (over)–parametrization of space of solution
  - induced initial data on a hypersurface
  - gauge: choice of hypersurface
- Dirac observables characterize solutions:
  F: phase space $\quad\rightarrow\quad$ **C**
  with F constant on (gauge) equivalent initial data

OBSERVABLES AND DIRAC OBSERVABLES



solution of EOM

${}^{B}P = ({}^{B}g, {}^{B}p)$

${}^{A}P = ({}^{A}g, {}^{A}p)$

gauge trafo

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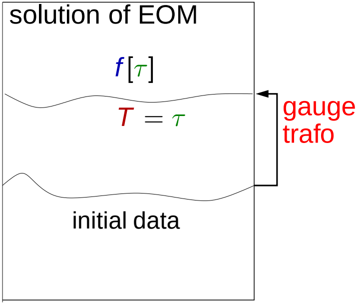
RELATIONAL OBSERVABLES (COMPLETE OBSERVABLES)

[ROVELLI ’90+, BD ’04+]



solution of EOM

$f[\tau]$

$T = \tau$

gauge trafo

initial data

- choose physical clock/time variables T: specify position and shape of a hypersurface by $T = \tau$
- ask a question f to this hypersurface

$f[\tau]$ gives value of phase space function f on hypersurface $T = \tau$

- characterizes solutions: is invariant under changes of initial data hypersurface
- compute this from arbitrary initial data satisfying constraints → Dirac observable

How to compute gauge invariant observables?

HOW TO COMPUTE DIRAC OBSERVABLES [BD '04+]

- find gauge trafo to hypersurface $T = \tau$, evaluate f on this hypersurface

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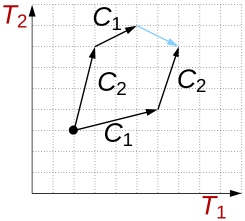
- find gauge trafo to hypersurface $T = \tau$

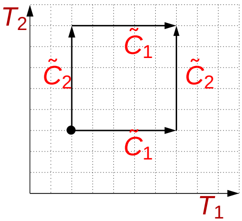
INTRODUCE CONVENIENT GAUGE GENERATORS

weakly Abelian constraints:

$$\tilde{C}_K(\sigma) = \int (A^{-1 j}{}_K(\sigma,\sigma')C_j(\sigma')d\sigma' , \qquad A_j^K(\sigma,\sigma') := \{T^K(\sigma), C_j(\sigma')\}$$

$$\Rightarrow \qquad \{T^K(\sigma), \tilde{C}(\sigma')_L\} \simeq \delta_L^K \delta(\sigma,\sigma')$$





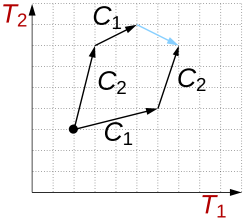
HOW TO COMPUTE DIRAC OBSERVABLES [BD '04+]

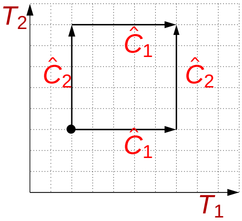
- find gauge trafo to hypersurface $T = \tau$

INTRODUCE CONVENIENT GAUGE GENERATORS

deparametrized form of constraints

$$\begin{aligned} \hat{C}_K &= P_K - h(T_k, q_a, p_b) \\ \Rightarrow & \quad \{T^K(\sigma), \tilde{C}(\sigma')_L\} = \delta^K_L \delta(\sigma, \sigma') \end{aligned}$$



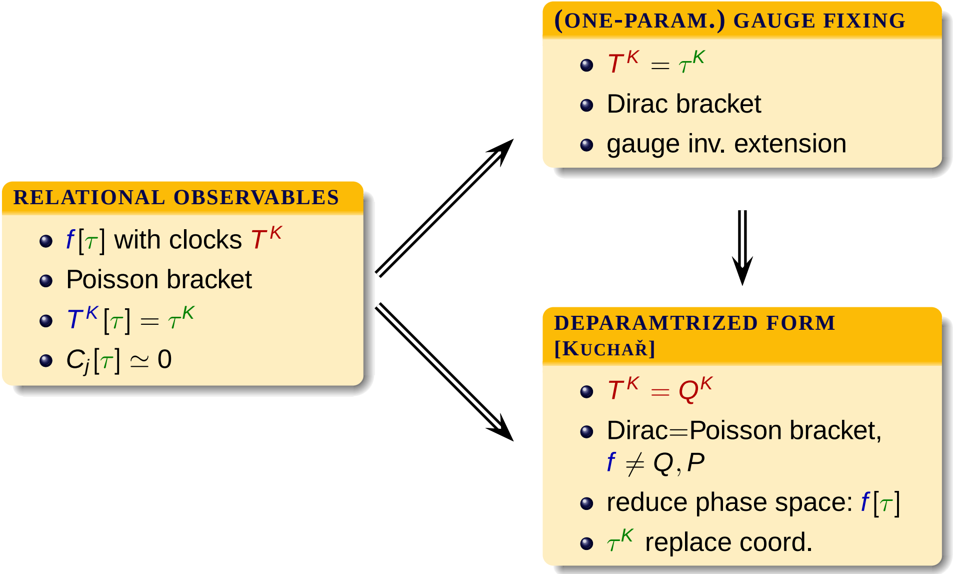


HOW TO COMPUTE DIRAC OBSERVABLES [BD '04+]

- find gauge trafo to hypersurface $T = \tau$, evaluate f on this hypersurface

$$f[\tau] \simeq \sum_{r=0}^{\infty} \frac{1}{r!} \int \{\cdots\{f, \tilde{C}_{K_1(\sigma_1)}\}, \cdots\}\tilde{C}_{K_r(\sigma_r)}\} \\ (\tau^{K_1} - T^{K_1})_{(\sigma_1)} \cdots (\tau^{K_r} - T^{K_r})_{(\sigma_r)} d\sigma_1 \ldots d\sigma_r$$

GAUGE FIX OR DEPARAMETRIZE [BD '04, THIEMANN '04]



RELATIONAL OBSERVABLES

- $f[\tau]$ with clocks T^K
- Poisson bracket
- $T^K[\tau] = \tau^K$
- $C_j[\tau] \simeq 0$

(ONE-PARAM.) GAUGE FIXING

- $T^K = \tau^K$
- Dirac bracket
- gauge inv. extension

DEPARAMTRIZED FORM [KUCHAŘ]

- $T^K = Q^K$
- Dirac=Poisson bracket, $f \neq Q, P$
- reduce phase space: $f[\tau]$
- τ^K replace coord.

HAMILTONIANS [THIEMANN '04],[BD, TAMBORNINO '07]

- generalize Hamiltonians: generate physical evolution in τ
- straightforward for deparametrized form [Kuchař '72]
- can also be defined for more general cases
- Hamiltonian depends on
  - choice of clocks
  - set of f's: for f Dirac observables the Hamiltonian is zero

- computational framework for relational observables available
- clear physical interpretation, in particular time evolution
- access phase space structure of observables

Can discuss structural properties of observables.

HOW MANY CLOCKS DO WE NEED?

- infinitely many constraints: $C_a(\sigma),\ a=1,2,3;\ \ C_\perp(\sigma)$
  $\sigma \in \Sigma$ (spatial hypersurface)

$\rightsquigarrow$ infinitely many clocks: $T^K(\sigma),\ K=0,\ldots 3,\ \ \sigma\in\Sigma$

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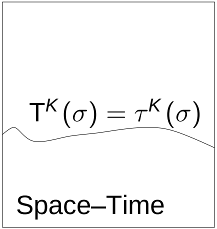

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- $f[\tau]$ = value of f on this embedding
- example: $f = \phi(\sigma^*)$

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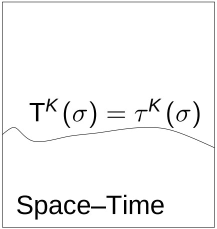

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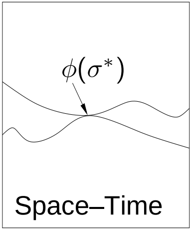
- infinitely many parameters needed to fix embedding, but $f = \phi(\sigma^*)$ does not depend on almost all aspects of the embedding

SPACE–TIME SCALARS



- condition for space–time scalar (K. Kuchař 1977): $\{C[N, N^a]\,,\,\phi(\sigma^*)\} = 0$ for $N(\sigma^*) = 0 = N^a(\sigma^*)$

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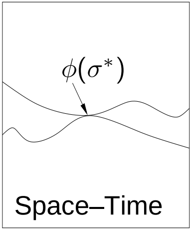

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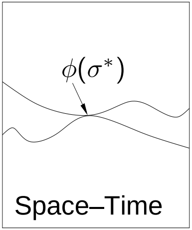

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! **Problem:** does not hold for $\{\{\phi(\sigma^*), C[M,M^a]\}\,, C[N,N^a]\ \}$

⇒ Choose as clocks space–time scalar fields $T^K(\sigma)$

⇒ $\tilde{C}_K(\sigma) = (A^{-1})^j_K(\sigma)C_j(\sigma)$ with $A^K_j(\sigma) = \{T^K(\sigma), C_j[1]\}$ are weakly Abelian

⇒ $\{\{\phi(\sigma^*), \tilde{C}[N^K]\}, \tilde{C}[N'^K]\} = 0$ for $N^K(\sigma^*) = 0 = N'^K(\sigma^*)$

REDUCING THE NUMBER OF CONSTRAINTS [BD '05]

NEED ONLY FOUR CONSTRAINTS FOR CALCULATION!

$$\phi(\sigma^*)[\tau] = \sum_{k_K=0}^{\infty} \frac{1}{|k|!} \tilde{C}_0^{k_0}[1] \cdots \tilde{C}_3^{k_3}[1] \, [\, \phi(\sigma^*) \,] \, (\tau_0 - T^0)^{k_0}(\sigma^*) \cdots (\tau_3 - T_3)^{k_3}(\sigma^*)$$

depends on only four parameters $\tau^K(\sigma^*)$

BUILDING SPACE–TIME SCALARS [BD '05]

Are there enough Space–Time Scalars?

- scalar matter fields, curvature invariants ⇝ physical coordinates
- express metric and derivatives in this coordinates
  - $\gamma^{KL} = T^K_\mu T^K_\nu \gamma^{\mu\nu} = T^K_{,a} T^L_{,b} g^{ab} - \{T^K, C_\perp[1]\}\{T^L, C_\perp[1]\}$
  - $\partial_M \gamma^{KL} := \{\gamma^{KL}, \tilde{C}_M[1]\}, \ldots$
- satisfy space–time scalar condition
- obtain (over–) complete set of Dirac observables (fix τ^0)

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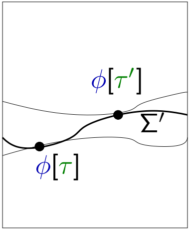
- "5–scalar proposal" works in the canonical formalism
- $\{\gamma^{KL}[\tau]\,,\ \gamma^{KL}[\tau']\}$ $\rightsquigarrow$ causality property, Green's functions
- connection between canonical and covariant picture!

CAUSALITY PROPERTIES



- assume clocks behave reasonable well
- consider phase space points where τ, τ' are spacelike related
- evaluate observables on Σ'

$\Rightarrow \{\phi[\tau], \phi[\tau']\} = 0$ on such phase space points

Spacelike related scalar observables commute.

The framework can be used to discuss structural properties of observables.

PERTURBATIONS AROUND SYMMETRY REDUCED SECTORS [BD, TAMBORNINO '06,'07]

Calculating Dirac observables is still very difficult.
It requires the solution of the dynamics of GR.

Consider approximations.

BACKGROUND INDEPENDENT QUANTUM GRAVITY

gauge invariant observables

⇙

large scale limit

MINISUPERSPACE APPROXIMATION

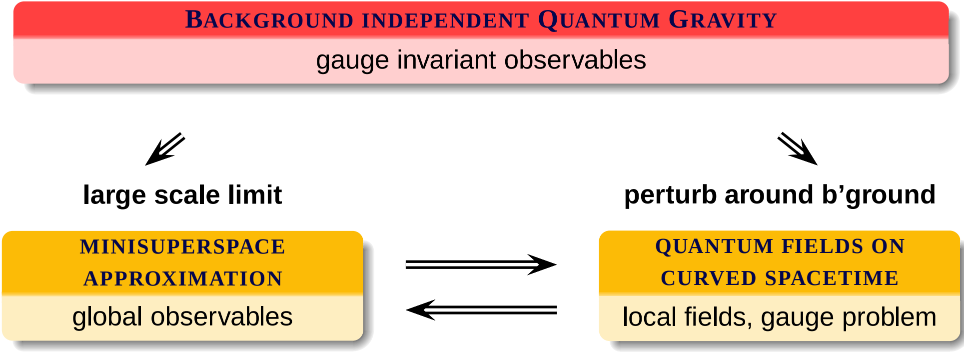
global observables

⇘

perturb around b'ground

QUANTUM FIELDS ON CURVED SPACETIME

local fields, gauge problem



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QUANTUM FIELDS ON CURVED SPACETIME

local fields, gauge problem

- quantum fields on quantum background?
- (quantum) dynamics of background influences dynamics of perturbations [Bojowald et al ’06+]
- backreaction of the (quantum) perturbations onto background

What are the gauge invariant observables reducing to local (quantum) field observables?

A GAUGE INVARIANT HAMILTONIAN FRAMEWORK

perturbations around symmetry reduced sectors (of gr):
cosmology, spherical, midisuper space

- **gauge invariant:**
  unambigious results, allows characterization of symmetric physical states
- **to arbitrary high order:**
  backreaction, scattering, embedding in full theory
- **canonical:**
  quantization, space–time algebra of observables: locality
- **“background” fully dynamical:**
  definition of physical time parameter, backreaction, effective approach

Do not perturb around fixed background metric but around phase space sector describing configurations with high symmetry.

SUBDIVIDE PHASE SPACE

phase space $=$
space of fields on Σ

$\mathcal{P} =$ projection acting on
phase space functions

$\Swarrow$ $\Searrow$

$\mathcal{P}$[functions]

- averaging

$$E = \frac{1}{3}\int_{\Sigma} E^a_j \delta^j_a \, d\sigma$$

$(\mathrm{Id} - \mathcal{P})$[functions]

- inhomogeneities

$$e^a_j = E^a_j - E\,\delta^a_j$$

HOMOGENEOUS AND INHOMOGENEOUS VARIABLES

- subdivision consistent with symplectic structure
- homogeneous variables arise through averaging from full phase space
- are fully dynamical → provide global clocks, important for invariance to higher order

---

- homogeneous variables (■ A, E) are zeroth order
- inhomogeneous (canonical) variables (■ a^j_a, e^b_k) are first order
- no higher order variables
- **higher order terms** are polynomials of first order variables

COMPUTE RELATIONAL OBSERVABLES

POWER SERIES EXPANSION

$$f[\tau] \simeq \sum_{r=0}^{\infty} \frac{1}{r!} \{\cdots\{ f\,, \tilde{C}_{K_1}\}, \cdots\}\tilde{C}_{K_r}\} \, (\tau^{K_1} - T^{K_1}) \cdots (\tau^{K_r} - T^{K_r})$$

EXPAND IN INHOMOGENEITIES

- for a certain set of clocks this power series expansion can be used to expand the relational observable **to arbitrary order m**
- results in **gauge invariant observables of order m**
- contact with standard perturbation theory: terms can be interpreted as
  - **free propagation**
  - **interaction terms**
  - **gauge invariant extension terms**

CLOCKS

- **first order clocks** (no zeroth order term) describe shape of hypersurface $T^K = \tau^K = 0$
  - longitudinal clocks: $T^0 = \Delta^{-1}(\frac{1}{2}{}^T a^d{}_d - {}^{LL}a^d{}_d)$ related to longitudinal gauge, non–local
  - scalar field (inhomogeneities) as clocks: $T^0 = \psi$ local description of hypersurface

- **global zeroth order clock** (no first order term) describes position of hypersurface, provide global time parameter, associated generator $\tilde{C}$ $T = \tau$
  - volume of hypersurface $T = \mathrm{Vol}_\Sigma$
  - averaged scalar field $T = \Psi$

These clocks allow for a consistent expansion of the relational observables for arbitrary values of τ.

PERTURBATIVE COMPLETE OBSERVABLES

reorder terms in power series expansion for $f[\tau]$

PERTURBATIVE COMPLETE OBSERVABLES

$$ {}^{(2)}f[\tau] \quad = \quad \alpha_{free}^{(\tau - T)}(f) $$ “free” propagation

PERTURBATIVE COMPLETE OBSERVABLES

$$
\begin{aligned}
{}^{(2)}f[\tau] \;=\;& \alpha_{\mathit{free}}^{(\tau - T)}(f) && \text{“free” propagation} \\
& - \sum_K {}^{(0)}\{\alpha_{\mathit{free}}^{(\tau - T)}(f), {}^{(1)}\tilde{C}_K\} T^K && \text{gauge invariant extension}
\end{aligned}
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Can be extended to any order.

REMARKS

- We can compute gauge invariant observables order by order.
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- [Giesel,Hofmann,Thiemann, Winkler '07]: coupling to dust, perturbations in deparametrized formalism

OUTLOOK: PERTURBATIVE FORMALISM

- discuss non–perturbative effects
  - perturb around flat background with accelerated observers
  - embed approximations into each other
- consider generalized boundary formulation in this framework

Is not having a good time exciting?

PROPERTIES OF OBSERVABLES

Properties of relational observables depend on

- choice of clocks (physical reference frame)
- dynamics of the system

■ volume of dust ball

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What kinds of clocks are available in GR?

We did not found any ("natural") perfect clocks yet.

THE PROBLEM OF CLOCKS

Strategies to deal with non–existence(?) of good clocks:

- coarse grain observables, adjust questions f
- perturb around sectors with good clocks
- ...

THE PROBLEM OF CLOCKS

Strategies to deal with non–existence(?) of good clocks:

A embrace it: derive limitations of knowledge as structural properties of GR [Giddings, Hartle, Marolf '05], [BD, Tambornino '06], [Arzano '07], [Girelli '07], [Aharonov et al '97], [Brunetti, Fredenhagen '01] . . .

B change dynamics (resulting in good(?) clocks)

- [Kuchař et al '90s]: Gaussian and harmonic reference fluid, dust, [Thiemann '06]: k–essence, [GHTW '07] : dust
- [Jacobson, Mattingly et al '01+]: Einstein–Aether theory
- [Milgrom]: MoND ⇝ [Bekenstein] TeVeS
- . . .

OPTION A

For scalar field clocks (standard matter: 4 percent of universe)

- $\{f[\tau], f[\tau+\epsilon]\} = G(\tau, \tau+\epsilon)\left(1 + \frac{\text{Energy}(f)}{\text{Energy(clocks)}}\right)$
- results in resolution limit: depends on energy of clocks
- cannot make energy of clocks arbitrarily large: black holes

$\rightsquigarrow$ (super–holographic) locality bound on number of degrees of freedom in a region [GHM '05]

- qm'al toy examples:
  - new kinds of uncertainty relations [Aharonov et al '97], [Brunetti, Fredenhagen '01] . . .
  - qm'al toy examples: cannot obtain self–adjoint operators
    → use POVMs instead ! [Brunetti, Fredenhagen '01]

OPTION B

change dynamics/ add good clocks

- partially motivated by dark matter problem
- introduce dynamically preferred frame
- hence Lorentz breaking
- previous argument on locality bound does not apply (there Hamiltonian for clocks is quadratic in momenta)
  $\Rightarrow$ fundamental difference to option A?
- are these clocks perfect? [Cotaldi, Wiseman, Withers '08]:
  - TeVeS has a unit timelike vector field
  - caustics for this vector field in gravitational wells
  - can avoid this by changing dynamics
- perfect clocks $\quad ? \leftrightarrow ? \quad$ existence of black holes

Discuss!