Gravitons from Spin Foams II:

a simplicial perspective

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Loop and Foams - Zakopane 2008

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Overview: see part I $\rightarrow$ Speziale's talk

Recover the graviton n-point correlation functions
from Loop Quantum Gravity $+$ a Spin Foam model

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- on a flat background
- gauge fixed n-point functions
  [but invariant $\sigma(hh \rightarrow hh)$]

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The strategy in this talk: separate the two issues

- an explicit calculation about (1) "at the single vertex level"
- a perspective on (2) (based on the calculation presented)

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1. From the Barrett-Crane model to perturbative Regge calculus
   - Computing spin correlations for the BC model
   - Comparison with perturbative area-Regge calculus
2. LQG and a test for the new spin-foam models
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Asymptotics of the Barrett-Crane model: some results

- Barrett-Crane JMP 1998: spin foam vertex for gravity $W_{\mathrm{BC}}(j_{mn}) = \sum_{i_n} \left(\{15j\}(j_{mn}, i_n)\right)^2$

  - starting point: some form of classical simplicial gravity $\longrightarrow$ "symbol" in $SU(2)$
  - by itself, not enough to guarantee simplicial gravity as a semiclassical regime

- Barrett-Williams ATMP 1999: large spin asymptotics

$$W_{\mathrm{BC}}(j_{mn}) \sim P(j_{mn}) \cos\left(S_{\mathrm{Regge}}(j_{mn}) + \frac{\pi}{4}\right) + D(j_{mn}) \qquad (*)$$

- Baez-Christensen-Egan CQG 2002, Barrett-Steele CQG 2003, Freidel-Louapre CQG 2003: the dominant contribution to the asymptotics comes from $D(j_{mn})$.

Notice that taking the large spin limit is not enough to identify *a* semiclassical regime: equivalent to large distance limit in the Hydrogen atom transition amplitude kernel $\langle \vec{x}_2 | U(T) | \vec{x}_1 \rangle$.

What is needed is a state peaked at large distance from the center and on *a* specific momentum:
- state $\psi_q(\vec{x})$ codes the initial conditions for a classical orbit
- the dynamics of the state identifies a Keplerian semiclassical regime $\longrightarrow$ $\langle \vec{x}(t_2) \vec{x}(t_1) \rangle_q$

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- B-Modesto NPB 2008: strenghten the relation with perturbative Regge calculus
  $\rightsquigarrow$ perturbative theory on a background from a theory without background

Computing spin correlations for the BC model

- B-Modesto NPB 2008: compute spin correlations on a semiclassical state for the Barrett-Crane model and recover from them *perturbative* quantum Regge-calculus

The strategy:

- $W_{\rm BC}$ as a sum of integrals, $W_{\rm BC}(j_{mn}) = \sum_k \int_{(S^3)^5} d\Omega \;\; f(\Omega) \;\; e^{\,i \sum j_{mn} Q^{(k)}_{mn}(\Omega)}$
- Avoid studying the $j_{mn} \gg 1$ asymptotics of $W_{\rm BC}(j_{mn}) \longrightarrow$ study directly the asymptotics of correlations on a (appropriately semiclassical) state peaked on large spins.

The technique:

- Introduce a generating functional for spin correlations:

$$< j_{mn}\, j_{m'n'} >_q = \frac{\partial}{\partial t_{mn}} \frac{\partial}{\partial t_{m'n'}} \log Z_q(t_{mn}) \Big|_{t=0}$$

- Study the large j_0 asymptotics of $Z_q(t_{mn})$

$$Z_q(t_{mn}) = \sum_{j_{mn}} W_{\rm BC}(j_{mn}) \; e^{i \sum j_{mn} t_{mn}} \; \Psi_q(j_{mn})$$

where:

- $q = (j_0, \phi_0) \qquad \longrightarrow \phi_0$ to be determined dynamically
- $\Psi_q(j_{mn}) = N e^{-\frac{1}{2} \sum \alpha_{mn\,m'n'} \frac{(j_{mn}-j_0)(j_{m'n'}-j_0)}{j_0}} \; e^{\,i \sum \phi_0 j_{mn}}$

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Details of the calculation. Two steps:

Identify the dominant contribution in $Z_q(t) = \sum_k \int d\Omega\, \Gamma^{(k)}(\Omega)$

- Sum over j: $\Gamma^{(k)}(\Omega) \sim \sum_j e^{ij(Q^{(k)}+t+\phi_0)} e^{-\frac{1}{2}\alpha\frac{(j-j_0)^2}{j_0}} \sim e^{-\frac{1}{2}j_0\alpha^{-1}(Q^{(k)}(\Omega)+t+\phi_0)^2}$
- Integrate $d\Omega$: saddle point present only if $\begin{cases} \phi_0 = \cos^{-1}(-1/4) \\ k = 0 \end{cases}$

Compute the dominant contribution, i.e. $k = 0$ and $\Omega \in D$

- Use *perturbative* stationary phase approximation for restricted BC integral:
  - asymptotic expansion in j_0^{-1}
  - perturbative expansion in $\delta j_{mn}/j_0 = O(1/\sqrt{j_0})$
- Result: $Z_q(t_{mn}) \approx \sum_{\delta j_{mn}} \mu_{j_0}(\delta j_{mn}) e^{iS_{j_0}(\delta j_{mn})} e^{i\sum \delta j_{mn} t_{mn}} \Psi_q(j_0 + \delta j_{mn})$

$$S_{j_0}(\delta j_{mn}) = \sum \phi_0 \delta j_{mn} + \frac{1}{2}\sum \frac{K_{(mn)(pq)}}{j_0}\delta j_{mn}\delta j_{pq} + \frac{1}{3!}\sum \frac{I_{(mn)(pq)(rs)}}{j_0^2}\delta j_{mn}\delta j_{pq}\delta j_{rs} + \dots$$

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$Z_q(t_{mn})$ non-trivial in t_{mn} only for specific ϕ_0

- $|Q(\Omega) + t + \phi_0| \lesssim 1/\sqrt{j_0}$ identifies domain $D \subset (S^3)^5$, $D =$ ball at Ω_t of radius $\sim 1/\sqrt{j_0}$
- Result:
  $$Z_q(t) = \int_D d\Omega\, \Gamma^{(0)}(\Omega) + B_q(t) = j_0^{-9/2} \mu(\Omega_t) e^{-\frac{1}{2} j_0 \alpha^{-1} \left(Q^{(0)}(\Omega_t)+t+\phi_0\right)^2} + O(j_0^{-11/2})$$

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The term which was dominant in the asymptotics of W_{BC} is subdominant in $Z_q(t)$.

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Details of the calculation. Two steps:

Identify the dominant contribution in $Z_q(t) = \sum_k \int d\Omega\, \Gamma^{(k)}(\Omega)$

- Sum over j: $\Gamma^{(k)}(\Omega) \sim \sum_j e^{ij(Q^{(k)}+t+\phi_0)} e^{-\frac{1}{2}\alpha \frac{(j-j_0)^2}{j_0}} \sim e^{-\frac{1}{2}j_0\alpha^{-1}(Q^{(k)}(\Omega)+t+\phi_0)^2}$
- Integrate $d\Omega$: saddle point present only if $\begin{cases} \phi_0 = \cos^{-1}(-1/4) \\ k = 0 \end{cases}$

$Z_q(t_{mn})$ non-trivial in t_{mn} only for specific ϕ_0

- $|Q(\Omega) + t + \phi_0| \lesssim 1/\sqrt{j_0}$ identifies domain $D \subset (S^3)^5$, $D =$ ball at Ω_t of radius $\sim 1/\sqrt{j_0}$
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$$Z_q(t) = \int_D d\Omega\, \Gamma^{(0)}(\Omega) + B_q(t) = j_0^{-9/2}\mu(\Omega_t) e^{-\frac{1}{2}j_0\alpha^{-1}\left(Q^{(0)}(\Omega_t)+t+\phi_0\right)^2} + O(j_0^{-11/2})$$

The term which was dominant in the asymptotics of W_{BC} is subdominant in $Z_q(t)$.

Compute the dominant contribution, i.e. $k = 0$ and $\Omega \in D$

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We have a perturbative theory on a background. What is it? $\rightsquigarrow$ perturbative Regge-calculus

Comparison with perturbative Regge calculus. I

At the classical level:

- Regge action for a single 4-simplex (≡ boundary term ≡ Hamilton function)

$$S_{\text{Regge}}(L_e) = \frac{1}{8\pi G_N} \sum_t A_t(L_e)(\pi - \theta_t(L_e))$$

- Expand around the regular 4-simplex, $S_{\text{Regge}}(L_0 + \delta L_e)$
- Area variables: $A_t = A_t(L_e)$ not invertible in general ($\longrightarrow$ Simone's talk)
- Area fluctuations are good variables perturbatively for a single regular 4-simplex

$$\delta L_e = \sum_e M_{et}(A_0)\delta A_t \quad \text{where} \quad M_{et} = \left(\frac{\partial A_t}{\partial L_e}\bigg|_{L_0} \right)^{-1}$$

- Perturbative *area*-Regge action: compute coefficients ϕ_0, $K_{t\,t'}$, $I_{t\,t' t''}$ ...

$$S_{A_0}(\delta A_t) = \frac{1}{8\pi G_N}\Big(\sum \phi_0 \delta A_t + \frac{1}{2}\sum \frac{K_{t\,t'}}{A_0}\delta A_t \delta A_{t'} + \frac{1}{3!}\sum \frac{I_{t\,t't''}}{A_0^2}\delta A_t \delta A_{t'}\delta A_{t''} + \dots \Big)$$

- $S_{A_0}(\delta A_t)$ coincides with $S_{j_0}(\delta j_{mn})$ once we identify

$$j_0 \equiv \frac{A_0}{8\pi G_N} \qquad \text{and} \qquad \delta j_{mn} \equiv \frac{\delta A_t}{8\pi G_N}$$

Comparison with perturbative Regge calculus. II

At the quantum level:

- In perturbative Regge calculus we can compute correlations of area fluctuations
- Generating functional $Z_0(T_t) = \int \prod d\delta A_t \; \mu_0(\delta A_t) \; e^{iS_0(\delta A_t)} \; e^{i\sum T_t \delta A_t} \; \psi_0(\delta A_t)$
- $\langle \delta A_t \rangle_0 = \frac{\partial}{\partial T_t} \log Z_0(T_t)\Big|_{T=0} = 0$ (consistency of the background)
- $\langle \delta A_t \, \delta A_{t'} \rangle_0 = 8\pi G_N \; A_0 (iK - \alpha)^{-1}_{t\,t'}$ (L_P^2 correction to background $A_0 A_0$)
- $\langle \delta A_t \, \delta A_{t'} \, \delta A_{t''} \rangle_{\text{conn}} = (8\pi G_N)^2 \; A_0 \sum I_{s\,s'\,s''} (iK - \alpha)^{-1}_{s\,t} (iK - \alpha)^{-1}_{s'\,t'} (iK - \alpha)^{-1}_{s''\,t''}$

Such perturbative correlations can be recovered non-perturbatively from BC. For instance:

$$\langle j_{mn} \, j_{rs} \rangle_q = j_0 \, j_0 \left(1 + \frac{(iK - \alpha)^{-1}_{r\,s}}{j_0} + O(1/j_0^2) \right)$$

- In perturbative area-Regge calculus correlations of fluctuations of angles, volumes . . . can be computed in terms of correlations of area-fluctuations. For instance for the volume $\delta V_{\text{tet}} = \sum Q_t^{(\text{tet})}(A_0) \delta A_t$
- Taking into account the correspondence between $W_{\text{BC}}(j_{mn})$ and perturbative area-Regge-calculus we have that

Correlations of volumes and angles can be computed for the Barrett-Crane model: they correspond to specific non-trivial functions of the spins j_{mn}.

- What is the relation of all this with LQG?

Gravitons from Spin Foams II

Outline of the talk

1. From the Barrett-Crane model to perturbative Regge calculus
   - Computing spin correlations for the BC model
   - Comparison with perturbative area-Regge calculus
2. LQG and a test for the new spin-foam models
3. Simplicial geometries, metrics, diffs and ... gravitons
4. Conclusions

LQG and a test for the new SF models

- Feature of the new Spin Foam models (EPR and FKLS): the boundary state space is the LQG one. $\rightsquigarrow$ $W_{\text{new}}(j_{mn}, i_n)$.

- Metric in Ashtekar variables: $h(x)h^{ab}(x) = \delta^{ij} E^a_i(x) E^b_j(x)$.
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- Compute geometry correlations on a semiclassical state

$$\langle \delta^{ij}\, \widehat{E_i(S_{mn})}\, \widehat{E_j(S_{mn'})} \;\; \delta^{kl}\, \widehat{E_k(S_{rs})}\, \widehat{E_l(S_{rs'})} \rangle_q =$$

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The gauge-fixing issue

- In quantum field theory the graviton propagator is the 2-point correlation function for a fluctuation of the metric in a given gauge.
- In quantum-Regge-calculus we can compute correlations of the geometry perturbatively.
- How do we compare the two? In which gauge?
- The same question can be asked for correlations of geometry computed non-perturbatively in LQG+SF. Where did we choose the gauge?

$\rightsquigarrow$ a perspective on the gauge-fixing issue in LQG+SF from the simplicial geometry point of view

Simplicial geometries and metrics

- Regge NC 1961: *General Relativity without coordinates*
- Friedberg-Lee NPB 1984: Regge Gravity *is* General Relativity with piecewise-flat metrics.

Geometry $\equiv$ $(\mathcal{M}, g_{\mu\nu}(x))$ up to diffeomorphisms

- in 2d the moduli space can be identified $\longrightarrow$ finite dimensional
- in 4d, if we restrict attention to simplicial geometries, the moduli space can be (almost) identified: $\begin{cases} \text{a skeleton } \mathcal{C} \\ \text{edge lenghts} \quad L_e \end{cases}$
- diffs: $g_{\mu\nu}^{\{\mathcal{C},L_e\}}(x) \quad \overset{\phi}{\rightarrow} \quad \left(\phi * g^{\{\mathcal{C},L_e\}}\right)_{\mu\nu}(x)$
- Einstein-Hilbert action $\equiv$ Regge action

  $S_{\text{EH}}[g_{\mu\nu}^{\{\mathcal{C},L_e\}}(x)] = S_{\text{Regge}}[\mathcal{C}, L_e]$

- Jevicki-Ninomiya PR 1986, Menotti-Peirano NPB 1995-1998: at the quantum level, requiring diff invariance fixes the functional measure $d\mu(\mathcal{C}, L_e)$ both perturbatively and non-perturbatively.

Some remarks [for fixed skeleton $\mathcal{C}$]

Avoid confusion between the following symmetries:

- Trasformations of edge lengths which leave the geometry invariant
  - redundacy of lenght variables when describing flat space
  - for fixed $\mathcal{C}$, the L_es are *almost* the moduli space
- Transformations of the edge lengths which leave the action invariant
  $S[L_e^0] = S[L_e^0 + \delta L_e^0] \ \ (+O(\delta L^3))$
  these are symmetries of the action, not of the geometry. Examples:
  - in 4d, $S[\lambda L_e^0] = \lambda^2 S[L_e^0]$ and for flat space $S[L_e^{\text{flat}}] = 0$

  $$\Rightarrow \quad \left. \frac{\partial^2 S}{\partial\, \delta L_e\, \partial\, \delta L_{e'}} \right|_{\text{flat}} \quad \text{has 1 zero mode}$$

  this is the zero mode we find in the Hessian from the BC model
  - discrete symmetries due to the choice of the skeleton $\mathcal{C}$

  [(i) and (ii) discussed in Rocek-Williams PLB 1981, Dittrich-Freidel-Speziale PRD 2007, Simone's talk]
- diffeomorphisms (actually not C^∞),

$$x^\mu \to \phi^\mu(x) \quad , \quad g^{\{\mathcal{C},L_e\}}(x) = \left(\phi * \bar{g}^{\{\mathcal{C},L_e\}}\right)_{\mu\nu}(x)$$

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Graviton propagator in terms of invariants

- Every gauge-fixed quantity is a specific (generally highly non-trivial) function of invariants
- The graviton propagator in a given gauge is a function of invariants - namely lengths in a simplicial setting.
- When we compute area correlators and claim that we are computing components of the graviton propagator, what do we mean? Is there a gauge F such that for instance $G^F_{(11)(22)} =< (A_1)^2 \, (A_2)^2 >$?

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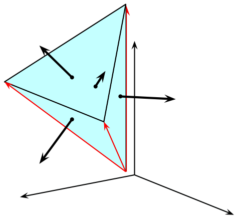
- Yes.

Coordinate choices for a simplicial 3-metric

- Metric in gauge F: $h^{\{L_e\}}_{ab}(x)\Big|_F = \left(\phi * \bar{h}^{\{L_e\}}\right)_{ab}(x)$
- It is possible to choose a cartesian coordinate system in each tetrahedron so that $\bar{h}^{\{L_e\}}_{ij}(y) = \delta_{ij}$ (curvature coded in transition functions)
- Different coordinate choice (adapted to the shape of the tetrahedron): choose edge vectors $e^i_M, \quad M = 1, 2, 3$ as basis

$$h_{MN} = \delta_{ij}\, e^i_M\, e^j_N$$

notice that: $h_{11} = L^2_{01}, \ldots$



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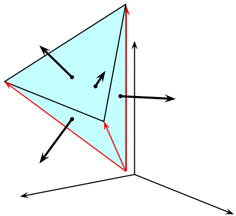

- Actually in LQG+SF we compute correlations of the density-1 inverse metric. With this coordinate choice it has the following components

$$h\, h^{MN} = \delta^{ij}\, \omega_i^M\, \omega_j^N = \left\{ \begin{array}{ccc} (A_1)^2 & A_1\, A_2\, \cos\theta_{12} & \cdot \\ A_1\, A_2\, \cos\theta_{12} & (A_2)^2 & \cdot \\ \cdot & \cdot & (A_3)^2 \end{array} \right\}$$

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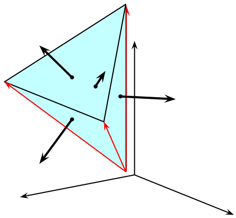

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- the entries of the matrix $h\, h^{MN}$ matches exactly with the geometric meaning of the operator $\delta^{ij}\, \widehat{E_i(S_{mn})}\, \widehat{E_j(S_{mn'})}$ in LQG.

Outline of the talk

1. From the Barrett-Crane model to perturbative Regge calculus
   - Computing spin correlations for the BC model
   - Comparison with perturbative area-Regge calculus
2. LQG and a test for the new spin-foam models
3. Simplicial geometries, metrics, diffs and ... gravitons
4. Conclusions

Conclusions

- The regime we are probing in our *simple* LQG+SF semiclassical calculations is a perturbative simplicial geometry regime
- We are computing correlations of diff-invariant quantities. Gauge-fixed *metric* correlations recovered as functions of such invariants

Two major open issues to address (before looking at the continuum):

- go beyond the “single-vertex level” but at fixed graph
- understand what happens when we start summing over two-complexes